Finding Normal Proportions
Sections 3.6, 3.7, 3.8

Lecture 10

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Outline

Notation

- We denote by $N(\mu, \sigma)$ the normal distribution with mean μ and standard deviation σ.

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- The number of sixes out of 720 rolls is $N(120, 10)$.

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- The number of heads out of 10,000 tosses is $N(5000, 50)$.
- The number of sixes out of 720 rolls is $N(120, 10)$.
- The IQ scores are $N(100, 15)$.

Outline

The Standard Normal Distribution

Definition (The Standard Normal Distribution)

The standard normal distribution is the normal distribution with $\mu = 0$ and $\sigma = 1$.

- Sketch the standard normal density curve.

The Standard Normal Distribution

The Standard Normal Distribution

If x is a variable with a normal distribution that has mean μ and standard deviation σ, then the variable z, defined by

$$z = \frac{x - \mu}{\sigma}$$

has the standard normal distribution.

Outline

Cumulative Proportions

Definition (Cumulative Proportion)

The cumulative proportion of a value x in a distribution is the proportion of the distribution that is less than or equal to x.

- The proportion is the same as the area under the curve (because the total area is 1).

Outline

Using the Standard Normal Table

- The Standard Normal Table is a table of cumulative normal proportions.
- It is found on pages 698 and 699.
- The values in the left margin are the first two significant digits of x.
- The values across the top are the third significant digit of x.
- The entry in that row and column is the cumulative proportion of x (the area to the left of x).

Using the Standard Normal Table

z	*0.09*	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*
–3.40	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.30	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
–3.20	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
–3.10	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
–3.00	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
–2.90	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
–2.80	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
–2.70	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
–2.60	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
–2.50	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
–2.40	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
–2.30	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
–2.20	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
–2.10	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
–2.00	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
–1.90	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
–1.80	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
–1.70	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
–1.60	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
–1.50	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
–1.40	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
–1.30	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
–1.20	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
–1.10	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
–1.00	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
–0.90	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
–0.80	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
–0.70	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
–0.60	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
–0.50	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
–0.40	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
–0.30	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
–0.20	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
–0.10	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
–0.00	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

Using the Standard Normal Table

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	O.7704	0.7734	O.7764	O.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9931
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get fewer than 5,100 heads?
- What fraction of the time will we get more than 4,950 heads?
- What fraction of the time will we get between 4,900 and 5,050 heads?

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get fewer than 5,100 heads?

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get fewer than 5,100 heads?
- We know (were told) that $\mu = 5000$ and that $\sigma = 50$.

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get fewer than 5,100 heads?
- We know (were told) that $\mu = 5000$ and that $\sigma = 50$.
- Find the z-score of 5100:

$$\begin{aligned} z\text{-score} &= \frac{5100 - 5000}{50} \\ &= \frac{100}{50} \\ &= 2.00. \end{aligned}$$

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get fewer than 5,100 heads?
- We know (were told) that $\mu = 5000$ and that $\sigma = 50$.
- Find the z-score of 5100:

$$\begin{aligned} z\text{-score} &= \frac{5100 - 5000}{50} \\ &= \frac{100}{50} \\ &= 2.00. \end{aligned}$$

- The table entry for 2.00 is 0.9772.

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get fewer than 5,100 heads?
- We know (were told) that $\mu = 5000$ and that $\sigma = 50$.
- Find the z-score of 5100:

$$\begin{aligned} z\text{-score} &= \frac{5100 - 5000}{50} \\ &= \frac{100}{50} \\ &= 2.00. \end{aligned}$$

- The table entry for 2.00 is 0.9772.
- We expect to get fewer than 5,100 heads about 97.72% of the time.

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get more than 4,950 heads?

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Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get more than 4,950 heads?
- Find the z-score of 4950:

$$\begin{aligned} z\text{-score} &= \frac{4950 - 5000}{50} \\ &= \frac{-50}{50} \\ &= -1.00. \end{aligned}$$

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get more than 4,950 heads?
- Find the z-score of 4950:

$$\begin{aligned} z\text{-score} &= \frac{4950 - 5000}{50} \\ &= \frac{-50}{50} \\ &= -1.00. \end{aligned}$$

- The table entry for -1.00 is 0.1587.

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get more than 4,950 heads?
- Find the z-score of 4950:

$$\begin{aligned} z\text{-score} &= \frac{4950 - 5000}{50} \\ &= \frac{-50}{50} \\ &= -1.00. \end{aligned}$$

- The table entry for -1.00 is 0.1587.
- We expect to get fewer than 4,950 heads about 15.87% of the time.

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get between 4,900 and 5,050 heads?

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get between 4,900 and 5,050 heads?
- Find the z-scores of 4900 and 5050:

$$z\text{-score of } 4000 = \frac{4900 - 5000}{50} = -2.00$$

$$z\text{-score of } 5500 = \frac{5050 - 5000}{50} = 1.00.$$

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get between 4,900 and 5,050 heads?
- Find the z-scores of 4900 and 5050:

$$z\text{-score of } 4000 = \frac{4900 - 5000}{50} = -2.00$$

$$z\text{-score of } 5500 = \frac{5050 - 5000}{50} = 1.00.$$

- The table entry for -2.00 is 0.0228 and the table entry for 1.00 is 0.8413.

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get between 4,900 and 5,050 heads?
- Find the z-scores of 4900 and 5050:

$$z\text{-score of } 4000 = \frac{4900 - 5000}{50} = -2.00$$

$$z\text{-score of } 5500 = \frac{5050 - 5000}{50} = 1.00.$$

- The table entry for -2.00 is 0.0228 and the table entry for 1.00 is 0.8413.
- The difference is $0.8413 - 0.0228 = 0.8185$.

Using the Normal Table

Example (Coin Tossing)

- If we toss a coin 10,000 times and do that repeatedly, what fraction of the time will we get between 4,900 and 5,050 heads?
- Find the z-scores of 4900 and 5050:

$$z\text{-score of } 4000 = \frac{4900 - 5000}{50} = -2.00$$

$$z\text{-score of } 5500 = \frac{5050 - 5000}{50} = 1.00.$$

- The table entry for -2.00 is 0.0228 and the table entry for 1.00 is 0.8413.
- The difference is $0.8413 - 0.0228 = 0.8185$.
- We expect to get between 4,900 and 5,050 heads about 81.85% of the time.

Outline

TI-83 - Standard Normal Areas

TI-83 Standard Normal Areas

- Press `2nd DISTR`.
- Select `normalcdf` (Item #2).
- Enter the lower and upper bounds of the interval.
  - If the interval is infinite to the left, enter `(-)E99` as the lower bound.
  - If the interval is infinite to the right, enter `E99` as the upper bound.
- Press `ENTER`. The area appears in the display.

Other Normal Curves

- If we are working with a different normal distribution, say $N(5000, 50)$, then how can we find areas under the curve?
- Use the same procedure as before, except enter the mean and standard deviation as the 3rd and 4th parameters of the `normalcdf` function.
- For example, the area between 4900 and 5050 is given by `normalcdf(4900,5050,5000,50)`.

Outline

Finding a Value from a Proportion

- Very often we know the proportion and we must find the value of x that gives that proportion.
- Typically, the proportion is in the upper tail or the lower tail.

Lower Tail

- If the proportion is in the lower tail, then we simply use the table "backwards."
  - Find the proportion in the body of the table.
  - Read across to the row heading and up to the column heading to get the value of x.

Lower Tail

- If the proportion is in the lower tail, then we simply use the table "backwards."
  - Find the proportion in the body of the table.
  - Read across to the row heading and up to the column heading to get the value of x.
- If the proportion is in the upper tail, then
  - First subtract it from 1 to get the area of the lower part.
  - Then proceed exactly as for a lower tail.

Example

Example (Bottom 20%)

- In the distribution $N(5000, 50)$, what value marks the 20th percentile (the lower 20%)?
- What value marks the top 10%?
- Where are the first and third quartiles?
- What two values mark the middle 10%?

TI-83 - Standard Normal Percentiles

TI-83 Standard Normal Percentiles

- Press `2nd DISTR`.
- Select `invnorm` (Item #3).
- Enter the lower area, as a decimal.
- If the distribution is not standard normal, then also enter
  - The mean
  - The standard deviation
- Press `ENTER`. The percentile appears in the display.

Outline

Assignment

Assignment

- Read Sections 3.6 - 3.8.
- Apply Your Knowledge: 8, 10, 11, 12.
- Exercises: 28, 30, 31, 34, 35, 37, 40, 43.